

Zitterbewegung, internal momentum and spin of the circular travelling wave electromagnetic electron

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Abstract

The study of this paper demonstrates that electron has Dirac delta like internal momentum (u, \vec{p}_θ) , going round in a circle of radius equal to half the reduced Compton wavelength of electron with tangential velocity c . The circular momentum \vec{p}_θ and energy u emanate from circular Dirac delta type rotating monochromatic electromagnetic (EM) wave that itself travels in another circle having radius equal to the reduced Compton wavelength of electron. The phenomenon of Zitterbewegung and the spin of electron are the natural consequences of the model. The spin is associated with the internal circulating momentum of electron in terms of four component spinor, which leads to the Dirac equation linking the EM electron model with quantum mechanical theory. Our model accurately explains the experimental results of electron channelling experiment, [P. Catillon et al., Found.Phys. 38, 659 (2008)], in which the momentum resonance is observed at $161.784 MeV/c$ corresponding to Zitterbewegung frequency of $80.874 MeV/c$ electron beam.

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I. INTRODUCTION

The electron is elementary particle, embodied with intrinsic properties such as charge, mass, spin and magnetic dipole moment. Historically, there are number of proposed extended classical models of electron [1–6] encompassing classical properties effectively. However, these models not only suffer from the electron instability problem but also are unable to explain the quantum mechanical (QM) behavior of electron. On the other hand, the celebrated QM model of Dirac [7] is an abstract mathematical model, which incorporates the intrinsic properties of electron in terms of Dirac matrices and 4-component wavefunction. Dirac's later works on extended model of electron [3, 4, 8] attempts to expound its classical properties. However, on the conceptual background the comprehension of the abstract point QM model and its possible association to electrons's properties is still not well clear.

While working on Dirac electron in an external electromagnetic field, Foldy-Wouthuysen noted that the particle spreads out over a region of dimension of the order of its Compton wavelength in space with its mean position $X' = x$. This behavior of the particle lead to the proposal of extended model of electron [9, 10]. Due to some reasons, the extended models or their interpretation such as the one given in [9] didn't get much attention. The detail studies on Dirac model by Schrödinger and others [11, 12] show that the electron undergoes an internal oscillatory motion having dimension of the order of Compton wavelength, called Zitterbewegung (Ztg). From the Dirac equation of motion for a free electron, Schrödinger showed that even in linear motion the electron executes an oscillatory motion with a frequency $\nu_o = 2m_o c/h$, where all the parameters bear their traditional meaning. He further concluded that the instantaneous velocity of a slowly moving electron is c .

Recently the electron model proposed by Alexander Burinskii [5, 13, 14] considers electron as a closed gravitational singular string, based on Kerr-Newman theory. The model is essentially in curved space time geometry. Similarly, the electron model [15] uses $(4 + 1)$ Ricci flat space time geometry to obtain exact solutions of the Maxwell's and gauged Dirac equations. These solutions are interpreted in terms of a geometric model of the electron with its spin. Hence these models are inherently geometric ones, based on curved or $(4 + 1)$ space-time, instead of a flat spacetime geometry. Again these models do not institute some physical mechanism(s) at the core level. Using the Maxwell's theory with the boundary condition of spherical conducting surface, Dirac presented an electron model [8], which explains the

existence of muon as the excited state of electron, however, it does not assign spin to the electron.

In a very recent study, a traveling wave electromagnetic (TWEM) model for electron is presented [16]. The model is based on rotating EM wave confined in a circle with radius equal to the reduced Compton wavelength (RCW) (\hbar/mc) of electron using Maxwell's theory as a physical mechanism at the core level. In TWEM model the electric field vector \vec{E}_r (a Dirac delta vector) rotates in the xy plane and the corresponding magnetic field vector \vec{H}_z is pseudo static Dirac delta vector along the z -direction. The EM wave traverses a circular trajectory with RCW of the electron in half space. The radius and energy of the modeled electron, matches precisely to the classical counterparts. The stability of EM electron is ensured by showing zero divergence of source-free EM energy-momentum stress tensor for the model. The charge generation mechanism has been discussed at length using gauge of EM fields. The time reversal in travelling wave corresponds to the reversal of \vec{E} and \vec{B} fields, which in turn gives rise to antiparticle (the positron) rotating EM wave. With four possible time-space ($\pm t, \pm \theta$) combinations we obtain four types of EM rotating waves. The overall picture at this level conforms the local $U(1)$ gauge symmetry. The monochromatic circulating EM wave, carry the corresponding energy-momentum in half the RCW of electron. The Ztg phenomenon arises naturally in this model as a consequence of circular flow of EM energy-momentum.

In this paper we use the TWEM model to demonstrate the electron's internal structure in terms of circular flow of energy-momentum (u_{em}, \vec{p}_θ) at velocity c . The two component wavefunction for energy-momentum, corresponding to left and right circular polarized wave(s) is shown to be related to the spin of electron, conforming to $SU(2)$ group symmetry. Furthermore, the negative and positive time circulating energy-momentum are shown to correspond to four component Dirac like spinor. In the rest frame, the four components of spinor satisfy massless Klein Gordon equation, which in turn leads to a massless Dirac or Weyl equation, governed by transformation rule under covering group $SL(2, C)$ of proper Lorentz group $SO^+(3, 1)$. Finally, the massive Dirac equation for TWEM electron has been obtained in boosted frame linking EM electron model with QM.

The internal dynamical structure of electron based on Ztg can be experimentally verified in line with experiments of electron channeling in silicon crystals [17, 18]. In electron channeling experiment, the momentum resonance is observed at twice the de Broglie frequency,

that is at $161.784\text{MeV}/c$ instead of $80.874\text{MeV}/c$. The authors describe it as a manifestation of de Broglie internal clock if squared amplitude is considered, that is, the Ztg frequency. In our proposed model the frequency of momentum is twice the de Broglie frequency of EM circulating wave. This is in good agreement with the phenomena observed in the electron channelling experiment.

II. ZITTERBEWEGUNG AND THE VELOCITY OF ELECTRON

In this section, for the paper to be self-contained, we present a brief review of the basic concepts of Ztg in electron. Although a number of different interpretations of Ztg of Dirac model are given [9, 19–27], however, we will briefly review the interpretation given in [11, 12]. The Dirac equation for spin-1/2 particle is

$$i\hbar\frac{\partial}{\partial t}\psi(r, t) = H\psi(r, t), \quad (1)$$

where H is the Hamiltonian for a free particle and can be written as

$$H = c\vec{\alpha} \cdot \vec{p} + \beta mc^2. \quad (2)$$

In Eq. (2) $\vec{\alpha}$ and β are the well known 4×4 Dirac matrices which satisfy following conditions,

$$\beta^2 = I; \quad \alpha_i \alpha_j + \alpha_j \alpha_i = 2\delta_{ij}I; \quad \alpha_i \beta + \beta \alpha_i = 0. \quad (3)$$

For a free electron, the velocity determined by Schrödinger [11], using Eq. (2) is

$$\vec{v} = c\vec{\alpha},$$

with c being the expectation value of \vec{v} , which means the electron is moving with an aggregate constant velocity c . It was further illustrated that \vec{v} can be expressed as.

$$\vec{v} = c^2 H^{-1} \vec{p} + c \vec{\eta}_o e^{-2iHt/\hbar}, \quad (4)$$

where \vec{p} is the linear momentum and $\vec{\eta}_o$ is a constant operator given by

$$\vec{\eta}_o = \vec{\eta}_o(0) = \vec{\alpha}(0) - cH^{-1}\vec{p}.$$

The velocity of electron in Eq. (4) consists of two parts, the regular part $c^2 H^{-1} \vec{p}$ and the oscillatory part $c \vec{\eta}_o e^{-2iHt/\hbar}$. The later part represents Ztg having amplitude equal to

$\hbar/2m_o c$ (half the RCW of electron) and angular frequency $\omega_z = 2m_o c^2/\hbar$. The oscillatory part, having extremely small amplitude, was believed unobservable directly [28]. However, a recent simulated work [29] on one dimensional Dirac dynamics for free particle shows that the Ztg is an observable phenomenon. Also, [30] proposes an experiment with ultracold atoms in a tripod level scheme on an optical lattice, to observe Ztg, at experimentally measurable frequencies. The *internal dynamics of 4 momentum* associated with the oscillatory part of electron is the source of Ztg of electron and is at the core of QM theory embedded in α_i and β of Dirac matrices.

III. INTERNAL 4-MOMENTUM OF CIRCULAR EM TRAVELLING WAVE ELECTRON

The relativistic momentum \vec{p}' of a free electron is given as

$$|\vec{p}'| = \pm \sqrt{\vec{p}^2 + \vec{p}_o^2}, \quad (5)$$

where \vec{p} is the linear momentum in x, y, z coordinates and $p_o = m_o c$ is the magnitude of momentum associated with the rest mass of electron. The momentum p_o is Lorentz invariant and behaves as a constant in the frame of the center of mass of the particle. However, there is dynamism of energy-momentum when electron is at rest, as noted by Hestenes [24] and others [31]. In this article we will consider the internal momentum dynamism (spin) in the center of mass frame of electron, which naturally arises in the TWEM model of electron [16]. The *EM* momentum of electron in the particle's rest frame $K_p(0)$ is given by

$$\vec{p}_o \equiv \vec{p}_\theta = \frac{1}{c} \vec{S}_\theta. \quad (6)$$

The angular momentum \vec{p}_θ is invariant momentum associated with the spin of electron. In the rest frame, we have $\vec{p} = 0$ in Eq. (5) and $p' = |\pm \vec{p}_\theta| = \left| \frac{1}{c} \vec{S}_\theta \right|$, which leads to the massless-light like case.

Using the internal momentum \vec{p}_θ and energy $u_{em} = (\varepsilon_o \vec{E}_r^2 + \mu_o \vec{H}_z^2)/2$ of electron in the rest state, we can define internal momentum of electron as follows

$$p_\theta^\mu = (p_\theta^o, \vec{p}_\theta), \quad (7)$$

where $p_\theta^o = u_{em}/c$. The p_θ^μ apparently forms 4-vector with p_θ^o , and \vec{p}_θ the time like and spatial component(s), respectively, in the rest frame of electron. It, however, does not transform

as 4-vector, since p_θ^μ is the internal degree of freedom of electron associated with the spin and subsist as a $2D$ object. In the rest frame of electron, p_θ^μ can be considered to dwell in wrapped coordinates within the particle [12, 32].

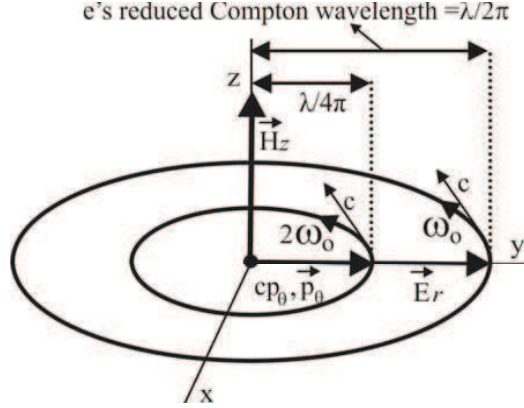


FIG. 1: The vector \vec{E}_r is circulating in a circle of radius equal to RCW and the corresponding momentum \vec{p}_θ and energy cp_θ are circulating in a circle of radius equal to $\frac{1}{2}$ RCW. The vectors \vec{E}_r and \vec{p}_θ have angular frequency ω_o and $2\omega_o$, respectively, but the same tangential velocity c .

Using the circular EM travelling wave expressions for TWEM model of electron, the \mathbf{p}_θ can be expressed as,

$$\vec{p}_\theta = \frac{1}{c} p'_o e^{\pm i/\hbar(-2u_{em}t \pm 2p_\theta r\theta)} \hat{\theta}, \quad (8)$$

where $p'_o = \frac{1}{c}[E_r^o H_z^o]$. We have a similar expression for the energy (time like) component $p_\theta^o = cp_\theta$. There are four EM rotating (circular polarized like) waves with circulating momentum given in Eq. (8). Two of these are associated with electron in positive (+ve) time (or +ve energy), right (\uparrow) $\equiv \psi^1$ and left circular (\downarrow) $\equiv \psi^2$ and other two associated with positron in negative (-ve) time (or -ve energy), right (\uparrow) $\equiv \psi^3$ and left circular (\downarrow) $\equiv \psi^4$. Therefore, we can define four spinors ψ^i ($i = 1, 2, 3, 4$) corresponding to four circulating polarized waves with four time-space ($\pm t, \pm \theta$) combinations.

Initially, we consider the case of ψ^1 (in $+t, +\theta$ coordinates) and will extend the idea at the end to encompass four component $\vec{\psi} \cong \psi^\alpha$; $\alpha = 1, 2, 3, 4$, to arrive at Dirac equation. The momentum in coordinate combination ($+t, +\theta$) is

$$\vec{p}_\theta = p'_o [e^{-i(\omega'_o t - k'_\theta \theta)}] \hat{\theta}, \quad (9)$$

with $\omega'_o = 2\omega_o$ and $k'_\theta = 2\omega_o r/c$. In this case the electric field vector \vec{E} ($+t, +\theta$) rotates with angular velocity ω_o , in a circle of radius $r_o \equiv \text{RCW}$. In Eq. (9), we note that the momentum \vec{p}_θ rotates with twice the angular velocity of \vec{E}_r vector, in a circle of radius $r_o/2$. From Eq. (9), we determine the angular velocity (or frequency) of energy-momentum as follows

$$\omega_\theta^p = \omega_\theta^u = \frac{\omega'_o}{k'_\theta} = \frac{c}{r_o}, \quad (10)$$

where ω_θ^p and ω_θ^u are the angular velocities of momentum and energy, respectively. The corresponding tangential velocities are $|\vec{v}^p| = |\vec{v}^u| = c$, (see Fig. (1)). The angular frequency ω'_o of energy-momentum is in conformity with Ztg angular frequency [11, 12].

As both the momentum \vec{p}_θ and the energy $p_\theta^o c$ are circulating with the velocity of light, hence, we will first consider massless Dirac or Weyl spinor within $(1/2)$ RCW of electron. In fact, the Schrödinger result for the expectation value of \vec{v} equals c for Ztg of electron is valid only if the electron (or its constituent spinors) behave as massless object(s) in the relevant region [6]. We will however see that with boost, the left and the right spinors combine through the mass term. This leads to the actual velocity of the particle in the moving frame and the spinors still circulate at c , the velocity of light.

The extended TWEM model of electron exhibits internal geometry in the form of rotating transverse EM wave in a circle of radius r_o (RCW). That is the source free, Dirac delta type EM fields exist in a sphere of radius r_o . The energy-momentum of EM travelling wave flows in a circle of radius $(1/2)r_o$ synchronously with EM wave in the xy plane with speed c , as shown in Fig.(1). The circulating energy-momentum subsist in $SU(2)$ symmetry group, bestows QM spin satisfying all $SU(2)$ commutation relations. As we show below, the magnitude of z -component of spin turns out to be $\hbar/2$ and the helicity corresponding to right circular or left circular states determine its spin up or down state, respectively. A similar geometry of extended model of electron based on Ztg, is described in [33].

IV. ELECTRON IN THE REST STATE

For electron in the rest state, Eq. (9) may be written as

$$\vec{p}_\theta = p_\theta e^{i\theta'} \hat{\theta}, \quad (11)$$

where $\theta' = (2m_or/\hbar)\theta$ and time exponential part is implicit. Using cylindrical coordinates, we can write

$$\vec{p}_\theta \equiv p_x \pm ip_y \quad (12)$$

This is the relation for momentum (and the corresponding energy), rotating, counterclockwise (ccw) and clockwise (cw) in xy plane, for $+$ and $-$ signs in Eq. (12), respectively. Then using the analogy of circular polarized waves as the combination of two orthogonal linear polarized waves with a phase difference of $\pm\pi/2$, we get

$$p_\theta e^{i\theta'} = p_o(\hat{x} \pm i\hat{y})e^{i\theta'}. \quad (13)$$

Similar expression can be obtained for the energy u_{em} . The energy-momentum taken together form four vector like object p_θ^μ . Now, we can write two component wave function in momentum space $\psi^\nu(\vec{p})$, for $\nu = 1, 2$,

$$\psi(\vec{p}) = \begin{bmatrix} \xi_R \\ \xi_L \end{bmatrix} e^{i\theta'} = \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}, \quad (14)$$

where the right and the left eigenfunctions are, respectively, given by $\xi_R \equiv (\xi_R^u, \xi_R^p)$ and $\xi_L \equiv (\xi_L^u, \xi_L^p)$. The superscripts u and p denote the energy and momentum parts, respectively. Moreover, the 4-component spinor $\psi^\alpha(\vec{p})$; $\alpha = 1, 2, 3, 4$ is obtained by including the $-ve$ time (or $-ve$ energy).

The four component wavefunction is the solution of the following second order wave equation in cylindrical coordinates [16],

$$\left[\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \psi^\alpha(\vec{p}) = 0. \quad (15)$$

This equation is analogous to massless Klein-Gordon equation in momentum space. Using the standard procedure for obtaining first order differential equation from second order, we get the massless Dirac equation for the spinor $\psi^\alpha(\vec{p})$

$$i\hbar \left[\gamma^o \frac{\partial}{\partial t} + \gamma^i \frac{1}{r} \frac{\partial}{\partial \theta} \right] \psi^\alpha(\vec{p}) = 0; \quad i = 1, 2, 3.$$

Writing γ^μ in Weyl representation

$$\gamma^o = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \quad \gamma = \begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix},$$

where I is 2×2 identity matrix and $\vec{\sigma}$ are the Pauli spin matrices. Using the units where $\hbar = c = 1$, we arrive at Weyl equations or massless Dirac equation in chiral representation

$$\begin{bmatrix} (E - \vec{p}_\theta \cdot \vec{\sigma}) \\ (E + \vec{p}_\theta \cdot \vec{\sigma}) \end{bmatrix} \begin{bmatrix} \vec{\psi}_R \\ \vec{\psi}_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (16)$$

Here wave factor $e^{i\theta'}$ is implicit and $\vec{\psi}_R$ and $\vec{\psi}_L$ are $\times 2$ block wavefunctions

$$\vec{\psi}_R \equiv \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}, \quad \text{and} \quad \vec{\psi}_L \equiv \begin{bmatrix} \psi^3 \\ \psi^4 \end{bmatrix}.$$

We notice that $\vec{\psi}_R$ and $\vec{\psi}_L$ are independent and there is no interference from one to the other in the rest state of electron.

Now, we define helicity operator for our case as

$$\hat{h}_\theta = \frac{\vec{\sigma} \cdot \vec{p}_\theta}{|\vec{p}_\theta|}. \quad (17)$$

The Weyl spinors are eigenstates of helicity operator $h = \vec{\sigma} \cdot \hat{n}$, with $\vec{p}/|\vec{p}| = \hat{n}$. The helicity is well defined for the massless Dirac fermion or Weyl fermion with eigenvalues equal to ± 1 . In this case we are considering only internal momentum \vec{p}_θ (2D object in the xy plane), therefore, eigenvalues of h_θ are $+1$ and -1 for ccw and cw rotation, respectively. Dirac massless case is also equivalent to ultra relativistic helicity (energy $E \gg m$) of particle (moving at velocity near to velocity of light c), again giving h_θ the eigenvalues ± 1 . The evidence for helicity (± 1) of massless Dirac electron is experimentally confirmed at Dirac point, such as in graphene [34, 35].

The intrinsic spin of electron is associated with the circular flow of energy-momentum ($p_\theta^o, \vec{p}_\theta$) of TWEM model. The spin angular momentum S is linked to the energy and angular frequency of the circularly polarized wavepacket as follows [36]

$$S \equiv u_{em}/\omega'.$$

With p_θ circulating in the xy -plane, it is straightforward to show by using $u_{em} = \hbar\omega_o$ and $\omega' = 2\omega_o$ that the z -component of spin is given by

$$S_z = \pm \frac{1}{2} \hbar. \quad (18)$$

where \pm stand for ccw and cw rotations, respectively. Alternatively, with the geometry of Fig. (1), we can determine the spin using the internal orbital angular momentum generated due to circular flow of energy-momentum

$$\vec{L}_{int} = \vec{r} \times (\vec{E}_r \times \vec{H}_z) = \vec{r} \times \vec{p}_\theta = |\vec{r}| |\vec{p}_\theta| \hat{\theta}.$$

Using all the known values, this leads to $L_{int} = \frac{1}{2}\hbar$. For free massless spinors, the spin eigenstates can also be taken as the eigenstates of helicity with eigenvalues ± 1 . Correspondingly, the z -components of spin have eigenvalues $\pm 1/2$, which results in Eq. (18). Similar results for spin emerging from Ztg, have been obtained in [33]. Also a topological structure of electron has been hypothesized in [6] and coinciding results for spin have been obtained for postulated Hubius Helix model of electron.

V. ELECTRON IN MOTION

To see the behavior of electron wavefunction in motion, we apply the Lorentz boost to Weyl spinors. The massless Weyl spinors seem to exist on the light cone, moving at the speed of light. Furthermore, the velocity derivative in proper time is zero, i.e. $\partial v / \partial \tau = 0$ at the speed of light. Applying boost to an object, already moving with speed of light is pointless. However in the TWEM electron energy-momentum spinor(s) is circulating around the origin (that is around the center of energy-momentum) at the speed of light, Eqs. (9, 10). Then, we can apply boost to center of energy-momentum frame, which we consider, initially at rest. The general Lorentz boost for spinors is given by [37]

$$S(\Lambda) = I \cosh \frac{\rho}{2} - \vec{\sigma} \cdot \hat{n} \sinh \frac{\rho}{2}, \quad (19)$$

where ρ is rapidity such that $\tanh \rho = \beta$ and \hat{n} is a unit vector in the direction of motion. Using the relations $\cosh \frac{\rho}{2} = [\frac{1}{2}(1 + \gamma)]^{\frac{1}{2}}$ and $\sinh \frac{\rho}{2} = [\frac{1}{2}(\gamma - 1)]^{\frac{1}{2}}$ along with the relativistic relations of energy $E = \gamma m$ and momentum $\vec{p} = \gamma m \vec{v} = E \vec{v}$, the Lorentz boost of Eq. (19) in terms of energy-momentum turns out to be

$$S(\Lambda) = \sqrt{\frac{E + m}{2m}} \begin{bmatrix} I + \frac{\vec{\sigma} \cdot \vec{p}}{E + m} & 0 \\ 0 & I - \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \end{bmatrix}. \quad (20)$$

Then we can write spinors $\psi_R(\vec{p})$ and $\psi_L(\vec{p})$ in the moving frame as

$$\begin{bmatrix} \psi_R(\vec{p}) \\ \psi_L(\vec{p}) \end{bmatrix} = S(\Lambda) \begin{bmatrix} \psi_R(0) \\ \psi_L(0) \end{bmatrix}, \quad (21)$$

where $\psi_R(0)$ and $\psi_L(0)$ are spinors in the rest frame. For $\psi_R(p)$ and $\psi_L(p)$, we get

$$\begin{bmatrix} \psi_R(\vec{p}) \\ \psi_L(\vec{p}) \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{E+m}{2m}} \left[I + \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \right] \\ \sqrt{\frac{E+m}{2m}} \left[I - \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \right] \end{bmatrix} \begin{bmatrix} \psi_R(0) \\ \psi_L(0) \end{bmatrix}.$$

Assuming $\psi_L(0) = \psi_R(0) = \psi(0)$, we obtain massive Dirac equation

$$\begin{bmatrix} E - \vec{p}_\theta \cdot \vec{\sigma} \\ E + \vec{p}_\theta \cdot \vec{\sigma} \end{bmatrix} \begin{bmatrix} \vec{\psi}_R \\ \vec{\psi}_L \end{bmatrix} = \begin{bmatrix} m\vec{\psi}_L \\ m\vec{\psi}_R \end{bmatrix}. \quad (22)$$

In the moving frame the left and the right spinors are now mixed through the mass term. Alternatively, we can mix up the left and the right spinors with an appropriate weightage and can obtain the mass property of the massless spinning (momentum) waves.

Now, for four component spinor $\vec{\psi}(\vec{p}) \equiv \psi^\alpha(\vec{p})$, we can write the massive Dirac electron equation in the momentum space

$$(\gamma^\mu p_\mu - m)\vec{\psi}(\vec{p}) = 0. \quad (23)$$

In Minkowski space, replacing $p_\mu \longrightarrow i\partial/\partial x^\mu = \partial_\mu$, $\mu = 0, 1, 2, 3$, the Dirac equation takes the form

$$(i\gamma^\mu \partial_\mu - m)\vec{\psi}(x) = 0. \quad (24)$$

Once the relation between energy-momentum of spinning EM fields with Dirac equation is established, we can associate all attributes of Dirac theory to the TWEM model of electron. The Dirac fields describing the model are found on the divergence free symmetric energy-momentum tensor $\Theta^{\alpha\beta}$ of the circulating EM wave which obeys conservation laws $\partial_\alpha \Theta^{\alpha 0} = \partial_\alpha \Theta^{\alpha i} = 0$ [16]. There are number of ways to determine spin from Dirac equations, however, we will follow [36] to show it for circular flow of energy-momentum of Dirac field. Using momentum density in Dirac fields, the angular momentum \vec{J} of electron (wave packet) becomes

$$\vec{J} = \frac{\hbar}{2i} \int \vec{x} \times [\psi^\dagger \vec{\nabla} \psi - (\vec{\nabla} \psi^\dagger) \psi] d^3x + \frac{\hbar}{2} \int (\psi^\dagger \vec{\Sigma} \psi) d^3x, \quad (25)$$

where

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix},$$

are Dirac spin matrices. In Eq. (25), the first part represents orbital angular momentum and the second part represents the spin. From the spin part of Eq. (25), the Dirac spin operator is obtained as

$$\vec{S} = \frac{\hbar}{2} \vec{\Sigma}.$$

The helicity operator \hat{h} of Dirac electron is projection of spin on linear momentum of electron, then

$$\hat{h} = \frac{\vec{\Sigma} \cdot \vec{p}}{2|\vec{p}|}.$$

Alexander Burinskii [5] has obtained similar results by considering Kerr singular ring (a closed gravitational string) for the electron model. He treats electron as electromagnetic excitation of the ring, forming, travelling wave along a string as the Kerr-Newman solution of Einstein-Maxwell's theory. The ordinary Dirac theory has been obtained from the solutions of the massless underlying theory, which regards Ztg as a corpuscular analogue of the travelling waves. The author attempts to establish relation between gravity and QM leading to Quantum Gravity. The Kerr singular ring model and TWEM model are vastly different in terms of their modeling background theories. The Kerr singular ring model is based on Kerr-Newman gravitational singular string whereas TWEM model is based on linear Maxwell's theory and quantization.

In TWEM electron model, the dynamism (due to circular motion) of energy-momentum gives rise to Ztg and associated properties, such as electron spin and helicity etc. This model pictures, the internal structure of electron (and positron) and interprets its underlying physical mechanism. In perspective of the above deliberations, we may infer that TWEM electron exhibits symmetry group $U(1)$ at traveling electromagnetic wave level [16]. The $2D$ (circulating momentum) object belongs to symmetry group $SU(2)$ in the rest frame as a massless Dirac spinor. Finally under boost and/or rotation, $SL(2, C)$ symmetry group is pertinent to spinors leading to massive Dirac theory. Hence electron is a complex object and exhibits blend of different symmetry groups at different levels of structure and with reference to frame of observation.

VI. CONCLUSION

In summary, we have used the TWEM model to explore the internal structure of electron. The circular traveling EM wave of the model carries, synchronous rotating energy-momentum (u, \vec{p}_θ) , in a circle of radius of half the reduced Compton wavelength of electron.

It is demonstrated that in TWEM model of electron, the Zitterbewegung and spin arise naturally as a result of circular motion of the energy-momentum of EM wave. Furthermore, the spinning energy-momentum wave function forms four component Dirac spinor like object corresponding to the four possible circular polarized like EM waves. For the four component energy-momentum spinor, the massless and massive Dirac equations follow directly in the rest frame and in the boosted frame, respectively. Alternatively, the four component spinor of the TWEM model is a solution to the Dirac equation. In this model, we show that the Dirac spinors are not just mathematical objects but are real objects with a physical mechanism(s) in the background. The TWEM electron model is blend of symmetry groups $U(1)$, $SU(2)$ and $SL(2, C)$, that is, different symmetries are relevant at different levels of internal structure and with reference to frame of observation.

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